Control Design for Interval Type-2 Fuzzy Systems Under Imperfect Premise Matching

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***Abstract*—This paper focuses on designing interval type-2 (IT2) control for nonlinear systems subject to parameter uncertainties. To facilitate the stability analysis and control synthesis, an IT2 Takagi–Sugeno (T–S) fuzzy model is employed to represent the dynamics of nonlinear systems of which the parameter uncertainties are captured by IT2 membership functions characterized by the lower and upper membership functions. A novel IT2 fuzzy controller is proposed to perform the control process, where the membership functions and number of rules can be freely chosen and different from those of the IT2 T–S fuzzy model. Consequently, the IT2 fuzzy-model-based (FMB) control system is with imperfectly matched membership functions, which hinders the stability analysis. To relax the stability analysis for this class of IT2 FMB control systems, the information of footprint of uncertainties and the lower and upper membership functions are taken into account for the stability analysis. Based on the Lyapunov stability theory, some stability conditions in terms of linear matrix inequalities are obtained to determine the system stability and achieve the control design. Finally, simulation and experimental examples are provided to demonstrate the effectiveness and the merit of the proposed approach.**

***Index Terms*—Fuzzy control, imperfect premise matching, interval type-2 (IT2) fuzzy control, stability analysis.**

I. Introduction

TYPE-1 fuzzy control approach has been successfully applied to a wide range of domestic and industrial control applications, which demonstrate that it is a promising control approach for complex nonlinear plants [1]–[4]. Stability analysis and control synthesis are the two main issues to be considered in the fuzzy control paradigm. It is well known that the Takagi–Sugeno (T–S) fuzzy model [5] (also known as the Takagi–Sugeno–Kang fuzzy model [6]) plays an important role to carry out stability analysis and control design [7]–[13], which provides a general modeling framework for nonlinear systems. The system dynamics of the nonlinear systems can be represented as an average weighted sum of some local linear subsystems, where the weightings are characterized by the type-1 membership functions.

Lyapunov stability theory is the most popular method to investigate the stability of type-1 fuzzy-model-based (FMB) control systems. Basic stability conditions in terms of linear matrix inequalities (LMIs) [14] were achieved in [15] and [16]. The FMB control system is guaranteed to be asymptotically stable if there exists a common solution to a set of Lyapunov inequalities in terms of LMIs. With the proposed parallel distributed compensation (PDC) design concept, some stability conditions were relaxed in [16]. More relaxed stability conditions under PDC can be found in [17]–[19]. With the consideration of the information of type-1 membership functions, stability conditions can be further relaxed [20]–[22]. Also, the fuzzy control concept was extended to other stability/control problems such as output feedback control [23], sampled-data control [26], control systems with time delay [8], [24], [25], tracking control [27], large-scale fuzzy systems [28], and even fuzzy neural networks [29].

Type-1 fuzzy sets are able to effectively capture the system nonlinearities but not the uncertainties. It has been shown in the literature that type-2 fuzzy sets [30], which extend the capability of type-1 fuzzy sets, are good in representing and capturing uncertainties, supported by a number of applications such as adaptive filtering [31], analog module implementation and design [32], [33], active suspension systems [34], autonomous mobiles [35], electrohydraulic servo systems [36], extended Kalman filters [37], dc–dc power converters [38], nonlinear control [39], [40], noise reduction [41], video streaming [42], and inverted pendulum control [43]. However, type-2 fuzzy set theory was developed for a general type-2 fuzzy logic system but not mainly for FMB control scheme. Consequently, there are few research studies about the type-2 FMB control systems in the literature. This motivates the investigation of the system stability and control design of type-2 FMB control systems.

Recently, some research has been done on system control and stability analysis based on the existing framework of type-2 fuzzy systems [39], [44]–[48]. In [31], a basic interval type-2 (IT2) T–S fuzzy model was proposed, which was extended to a more general IT2 T–S fuzzy model [39] for a wider class of nonlinear systems suitable for system analysis and control design. Preliminary stability analysis work on IT2 FMB system can be found in [39] and [48] of which a set of LMI-based

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stability conditions was obtained determining the system stability and facilitating the control synthesis.

In this paper, we investigate the stability of IT2 FMB control systems under imperfect premise matching. Unlike the authors' work in [39] under the PDC design concept, it was required that the IT2 fuzzy controller shares the same premise membership functions and the same number of rules as those of the IT2 T–S fuzzy model. These limitations constrain the design flexibility and increase the implementation complexity of the IT2 fuzzy controller. The work in this paper eliminates these limitations by proposing an IT2 fuzzy controller in which the membership functions and the number of rules can be freely chosen, enhancing the applicability of the IT2 FMB control scheme. By choosing simple membership functions and a smaller number of rules, it can reduce the implementation complexity of the IT2 fuzzy controller, resulting in a lower implementation cost. However, the IT2 FMB control systems will have imperfectly matched membership functions, potentially leading to more difficult stability analysis as the favorable property of the PDC design concept vanishes.

To carry out the stability analysis for IT2 FMB control system subject to imperfect premise membership functions, the lower and upper membership functions characterized by the footprint of uncertainty (FOU) are chosen to be a favorable representation. This favorable representation allows the lower and upper membership functions to be taken in the stability analysis. Consequently, the stability conditions in terms of LMIs are membership function dependent, which is applied to the nonlinear plant under consideration, but not a family considered in some existing work. The preliminary result of the authors in [48] provides technical support to the work in this paper. To further relax the stability conditions, the FOU is divided into a number of sub-FOUs. The information of the sub-FOUs, along with those of lower and upper membership functions, is brought to the stability analysis. Based on the Lyapunov stability theory, LMI-based stability conditions are obtained to guarantee the stability of the IT2 FMB control systems and synthesize the IT2 fuzzy controller.

The organization of this paper is as follows. In Section II, the IT2 T–S fuzzy model representing the nonlinear plant subject to parameter uncertainties, the IT2 fuzzy controller, and the IT2 FMB control systems are presented. In Section III, LMI-based stability conditions are obtained based on the Lyapunov stability theory for the IT2 FMB control systems. In Section IV, simulation and experimental examples are given to illustrate the merits of the proposed IT2 FMB control scheme. In Section V, a conclusion is drawn.

II. Preliminaries

Considering a nonlinear plant subject to parameter uncertainties represented by an IT2 T–S fuzzy model [31], [39], an IT2 fuzzy controller is proposed to perform the control process. An IT2 FMB control system is formed by connecting the IT2 T–S fuzzy model and the IT2 fuzzy controller in a closed loop. In this paper, it is not required that both the IT2 T–S fuzzy model and the IT2 fuzzy controller share the same premise membership functions and the same number of rules.

A. IT2 T–S Fuzzy Model

A p-rule IT2 T–S fuzzy model [31], [39] is employed to describe the dynamics of the nonlinear plant. The rule is of the following format where the antecedent contains IT2 fuzzy sets and the consequent is a linear dynamical system:

$$\text{Rule } i: \text{IF } f_1(\mathbf{x}(t)) \text{ is } \tilde{M}_1^i \text{ AND} \cdots \text{AND } f_\Psi(\mathbf{x}(t)) \text{ is } \tilde{M}_\Psi^i \\ \text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) \quad (1)$$

where $\tilde{M}_\alpha^i$ is an IT2 fuzzy set of rule i corresponding to the function $f_\alpha(\mathbf{x}(t))$, $\alpha = 1, 2, \ldots, \Psi$ and $i = 1, 2, \ldots, p$; Ψ is a positive integer; $\mathbf{x}(t) \in \Re^n$ is the system state vector; $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times m}$ are the known system and input matrices, respectively; and $\mathbf{u}(t) \in \Re^m$ is the input vector. The firing strength of the ith rule is of the following interval sets:

$$W_i(\mathbf{x}(t)) = [\underline{w}_i(\mathbf{x}(t)), \quad \overline{w}_i(\mathbf{x}(t))], \qquad i = 1, 2, \ldots, p \quad (2)$$

where

$$\underline{w}_i(\mathbf{x}(t)) = \prod_{\alpha=1}^{\Psi} \underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t))) \geq 0 \quad (3)$$

$$\overline{w}_i(\mathbf{x}(t)) = \prod_{\alpha=1}^{\Psi} \overline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t))) \geq 0 \quad (4)$$

$$\overline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t))) \geq \underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t))) \geq 0 \quad (5)$$

$$\overline{w}_i(\mathbf{x}(t)) \geq \underline{w}_i(\mathbf{x}(t)) \geq 0 \qquad \forall i \quad (6)$$

in which $\underline{w}_i(\mathbf{x}(t))$, $\overline{w}_i(\mathbf{x}(t))$, $\underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t)))$, and $\overline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t)))$ denote the lower grade of membership, upper grade of membership, lower membership function, and upper membership function, respectively. The inferred IT2 T–S fuzzy model [39] is defined as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} \tilde{w}_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)) \quad (7)$$

where

$$\tilde{w}_i(\mathbf{x}(t)) = \underline{\alpha}_i(\mathbf{x}(t))\underline{w}_i(\mathbf{x}(t)) + \overline{\alpha}_i(\mathbf{x}(t))\overline{w}_i(\mathbf{x}(t)) \geq 0 \qquad \forall i \quad (8)$$

$$\sum_{i=1}^{p} \tilde{w}_i(\mathbf{x}(t)) = 1 \quad (9)$$

$$0 \leq \underline{\alpha}_i(\mathbf{x}(t)) \leq 1 \qquad \forall i \quad (10)$$

$$0 \leq \overline{\alpha}_i(\mathbf{x}(t)) \leq 1 \qquad \forall i \quad (11)$$

$$\underline{\alpha}_i(\mathbf{x}(t)) + \overline{\alpha}_i(\mathbf{x}(t)) = 1 \qquad \forall i \quad (12)$$

in which $\underline{\alpha}_i(\mathbf{x}(t))$ and $\overline{\alpha}_i(\mathbf{x}(t))$ are nonlinear functions not necessarily be known but exist, $\tilde{w}_i(\mathbf{x}(t))$ can be regarded as the grades of membership of the embedded membership functions, and (8) defines the type reduction.

Remark 1: It can be seen from (9) that the actual grades of membership $\tilde{w}_i(\mathbf{x}(t))$ can be reconstructed and expressed as a linear combination of $\underline{w}_i(\mathbf{x}(t))$ and $\overline{w}_i(\mathbf{x}(t))$, characterized by the lower and upper membership functions $\underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t)))$ and $\overline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t)))$, which are scaled by the nonlinear functions $\underline{\alpha}_i(\mathbf{x}(t))$ and $\overline{\alpha}_i(\mathbf{x}(t))$, respectively. In other words, any membership functions within the FOU [39] can be reconstructed by the lower and upper membership functions. As the nonlinear plant is subject to parameter uncertainties, $\tilde{w}_i(\mathbf{x}(t))$ will depend on the parameter uncertainties, thus leading to the values of $\underline{\alpha}_i(\mathbf{x}(t))$ and $\overline{\alpha}_i(\mathbf{x}(t))$ to be uncertain. It should be noted that the IT2 T–S fuzzy model (7) serves as a mathematical tool to facilitate the stability analysis and control synthesis and is not necessarily implemented.

B. IT2 Fuzzy Controller

An IT2 fuzzy controller with c rules of the following format is proposed to stabilize the nonlinear plant represented by the IT2 T–S fuzzy model (7):

$$\text{Rule } j \text{ IF } g_1(\mathbf{x}(t)) \text{ is } \tilde{N}_1^j \text{ AND} \cdots \text{ AND } g_\Omega(\mathbf{x}(t)) \text{ is } \tilde{N}_\Omega^j \text{ THEN } \mathbf{u}(t) = \mathbf{G}_j\mathbf{x}(t) \quad (13)$$

where $\tilde{N}_\beta^j$ is an IT2 fuzzy set of rule j corresponding to the function $g_\beta(\mathbf{x}(t))$, $\beta = 1, 2, \ldots, \Omega$ and $j = 1, 2, \ldots, c$; Ω is a positive integer; and $\mathbf{G}_j \in \Re^{m\times n}$, $j = 1, 2, \ldots, c$, represents the constant feedback gains to be determined. The firing strength of the jth rule is the following interval sets:

$$M_j(\mathbf{x}(t)) = \left[\underline{m}_j(\mathbf{x}(t)), \quad \overline{m}_j(\mathbf{x}(t))\right], \qquad j = 1, 2, \ldots, c \quad (14)$$

where

$$\underline{m}_j(\mathbf{x}(t)) = \prod_{\beta=1}^{\Omega} \underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t))) \geq 0 \quad (15)$$

$$\overline{m}_j(\mathbf{x}(t)) = \prod_{\beta=1}^{\Omega} \overline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t))) \geq 0 \quad (16)$$

$$\overline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t))) \geq \underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t))) \geq 0 \qquad \forall j \quad (17)$$

in which $\underline{m}_j(\mathbf{x}(t))$, $\overline{m}_j(\mathbf{x}(t))$, $\underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t)))$, and $\overline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t)))$ stand for the lower grade of membership, upper grade of membership, lower membership function, and upper membership function, respectively. The inferred IT2 fuzzy controller is defined as follows:

$$\mathbf{u}(t) = \sum_{j=1}^{c} \tilde{m}_j(\mathbf{x}(t))\mathbf{G}_j\mathbf{x}(t) \quad (18)$$

where

$$\tilde{m}_j(\mathbf{x}(t)) = \frac{\underline{\beta}_j(\mathbf{x}(t))\,\underline{m}_j(\mathbf{x}(t)) + \overline{\beta}_j(\mathbf{x}(t))\,\overline{m}_j(\mathbf{x}(t))}{\sum_{k=1}^{c}\left(\underline{\beta}_k(\mathbf{x}(t))\,\underline{m}_k(\mathbf{x}(t)) + \overline{\beta}_k(\mathbf{x}(t))\,\overline{m}_k(\mathbf{x}(t))\right)} \geq 0 \qquad \forall j \quad (19)$$

$$\sum_{j=1}^{c} \tilde{m}_i(\mathbf{x}(t)) = 1 \quad (20)$$

$$0 \leq \underline{\beta}_j(\mathbf{x}(t)) \leq 1 \qquad \forall j \quad (21)$$

$$0 \leq \overline{\beta}_j(\mathbf{x}(t)) \leq 1 \qquad \forall j \quad (22)$$

$$\underline{\beta}_j(\mathbf{x}(t)) + \overline{\beta}_j(\mathbf{x}(t)) = 1 \qquad \forall j \quad (23)$$

in which $\underline{\beta}_j(\mathbf{x}(t))$ and $\overline{\beta}_j(\mathbf{x}(t))$ are predefined functions, $\tilde{m}_j(\mathbf{x}(t))$ can be regarded as the grades of membership of the embedded membership functions, and (19) is the type reduction.

Remark 2: Compared with the IT2 fuzzy controller in [39], the proposed one in (18) has the following two enhancements.

1) The type reduction for the IT2 fuzzy controller in [39] is characterized by the average normalized membership grades of the lower and upper membership functions, e.g., $\underline{\beta}_j(\mathbf{x}(t)) = \overline{\beta}_j(\mathbf{x}(t)) = 0.5$ for all j. In this paper, the type reduction of the proposed IT2 fuzzy controller (18) is characterized by two predefined functions $\underline{\beta}_j(\mathbf{x}(t))$ and $\overline{\beta}_j(\mathbf{x}(t))$.
2) The proposed IT2 fuzzy controller (18) does not need to share the same lower and upper premise membership functions and the same number of fuzzy rules as those of the IT2 T–S fuzzy model (7). These two enhancements offer a higher design flexibility to the IT2 fuzzy controller. Moreover, by employing simple membership functions and a smaller number of fuzzy rules, the implementation complexity of the IT2 fuzzy controller (18) can be reduced.

C. IT2 FMB Control Systems

From (7) and (18), with the property of $\sum_{i=1}^{p}\tilde{w}_i(\mathbf{x}(t)) = \sum_{j=1}^{c}\tilde{m}_j(\mathbf{x}(t)) = \sum_{i=1}^{p}\sum_{j=1}^{c}\tilde{w}_i(\mathbf{x}(t))\tilde{m}_j(\mathbf{x}(t)) = 1$, we have the following IT2 FMB control system:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \sum_{i=1}^{p}\tilde{w}_i(\mathbf{x}(t))\left(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\sum_{j=1}^{c}\tilde{m}_j(\mathbf{x}(t))\mathbf{G}_j\mathbf{x}(t)\right)\\ &= \sum_{i=1}^{p}\sum_{j=1}^{c}\tilde{w}_i(\mathbf{x}(t))\,\tilde{m}_j(\mathbf{x}(t))\,(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j)\,\mathbf{x}(t). \end{aligned} \quad (24)$$

The control objective of this paper is to guarantee the system stability by determining the feedback gains $\mathbf{G}_j$, such that the IT2 fuzzy controller (18) is able to drive the system states to the origin, i.e., $\mathbf{x}(t) \to \mathbf{0}$, as time $t \to \infty$.

Basic LMI-based stability conditions guaranteeing the stability of the FMB control system in the form of (24) are given in the following theorem.

Theorem 1 [15]: The FMB control system in the form of (24) is guaranteed to be asymptotically stable if there exist matrices $\mathbf{N}_j \in \Re^{m\times n}$, $j = 1, 2, \ldots, c$, and $\mathbf{X} = \mathbf{X}^{\mathrm{T}} \in \Re^{n\times n}$ such that the following LMIs are satisfied:

$$\mathbf{X} > 0$$

$$\mathbf{Q}_{ij} = \mathbf{A}_i\mathbf{X} + \mathbf{X}\mathbf{A}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{N}_j + \mathbf{N}_j^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} < 0 \qquad \forall i, j$$

where the feedback gains are defined as $\mathbf{G}_j = \mathbf{N}_j\mathbf{X}^{-1}$ for all j.

Remark 3: The stability conditions in Theorem 1 are very conservative as the membership functions of both the fuzzy model and fuzzy controller are not considered. The stability conditions can be reduced to $\mathbf{Q}_{ij} = \mathbf{A}_i\mathbf{X} + \mathbf{X}\mathbf{A}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{N} + \mathbf{N}^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} < 0$ for all i by choosing a common feedback gain, i.e., $\mathbf{N} = \mathbf{N}_j$ for all j, resulting in a linear controller.

To facilitate the stability analysis of the IT2 FMB control system (24), the state space of interest denoted as Φ is divided into q connected substate spaces denoted as Φ_k, $k = 1, 2, \ldots, q$, such that $\Phi = \bigcup_{k=1}^{q} \Phi_k$. Furthermore, to consider more information of the IT2 membership functions, local lower and upper membership functions within the FOU are introduced. Considering the FOU being divided into $\tau + 1$ sub-FOUs, in the lth sub-FOU, $l = 1, 2, \ldots, \tau + 1$, the lower and upper membership functions are defined as follows:

$$\underline{h}_{ijl}(\mathbf{x}(t)) = \sum_{k=1}^{q}\sum_{i_1=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{r=1}^{n} v_{ri_rkl}(x_r(t)) \times \underline{\delta}_{iji_1i_2\cdots i_nkl} \quad \forall\, i, j, k, l \tag{25}$$

$$\overline{h}_{ijl}(\mathbf{x}(t)) = \sum_{k=1}^{q}\sum_{i_1=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{r=1}^{n} v_{ri_rkl}(x_r(t)) \times \overline{\delta}_{iji_1i_2\cdots i_nkl} \quad \forall\, i, j, k, l \tag{26}$$

$$0 \le \underline{h}_{ijl}(\mathbf{x}(t)) \le \overline{h}_{ijl}(\mathbf{x}(t)) \le 1 \tag{27}$$

$$0 \le \underline{\delta}_{iji_1i_2\cdots i_nkl} \le \overline{\delta}_{iji_1i_2\cdots i_nkl} \le 1 \tag{28}$$

where $\underline{\delta}_{iji_1i_2\cdots i_nkl}$ and $\overline{\delta}_{iji_1i_2\cdots i_nkl}$ are constant scalars to be determined; $0 \le v_{ri_skl}(x_r(t)) \le 1$ and $v_{r1kl}(x_r(t)) + v_{r2kl}(x_r(t)) = 1$ for $r, s = 1, 2, \ldots, n$; $l = 1, 2, \ldots, \tau + 1$; $i_r = 1, 2$; $\mathbf{x}(t) \in \Phi_k$; and $v_{ri_sk}(x_r(t)) = 0$ if otherwise. As a result, we have $\sum_{k=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{r=1}^{n} v_{ri_rkl}(x_r(t)) = 1$ for all l, which is used in the stability analysis.

We then express the IT2 FMB control system (24) in the following favorable form:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c}\tilde{h}_{ij}(\mathbf{x}(t))(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j)\mathbf{x}(t) \tag{29}$$

where

$$\begin{aligned}\tilde{h}_{ij}(\mathbf{x}(t)) &\equiv \tilde{w}_i(\mathbf{x}(t))\,\tilde{m}_j(\mathbf{x}(t)) \\ &= \sum_{l=1}^{\tau+1}\xi_{ijl}(\mathbf{x}(t))\big(\underline{\gamma}_{ijl}(\mathbf{x}(t))\,\underline{h}_{ijl}(\mathbf{x}(t)) \\ &\quad + \overline{\gamma}_{ijl}\overline{h}_{ijl}(\mathbf{x}(t))\big) \qquad \forall\, i, j\end{aligned} \tag{30}$$

with

$$\sum_{i=1}^{p}\sum_{j=1}^{c}\tilde{h}_{ij}(\mathbf{x}(t)) = 1. \tag{31}$$

$0 \le \underline{\gamma}_{ijl}(\mathbf{x}(t)) \le \overline{\gamma}_{ijl}(\mathbf{x}(t)) \le 1$ has two functions, which are not necessary to be known, exhibiting the property that $\underline{\gamma}_{ijl}(\mathbf{x}(t)) + \overline{\gamma}_{ijl}(\mathbf{x}(t)) = 1$ for all i, j, and l. $\xi_{ijl}(\mathbf{x}(t)) = 1$ if the membership function $h_{ijl}(\mathbf{x}(t))$ is within the sub-FOU l; otherwise, $\xi_{ijl}(\mathbf{x}(t)) = 0$.

Remark 4: It should be noted that only one $\xi_{ijl}(\mathbf{x}(t)) = 1$ among the $\tau + 1$ sub-FOUs at any time instant and the rest equal zero for the ijth membership function $\tilde{h}_{ij}(\mathbf{x}(t))$. It can be seen from (30) that, the more the sub-FOUs are considered, the more the information about the FOU is contained in the local lower and upper membership functions.

Remark 5: The local lower and upper membership functions can reconstruct $\tilde{h}_{ij}(\mathbf{x}(t)) \equiv \tilde{w}_i(\mathbf{x}(t))\tilde{m}_j(\mathbf{x}(t))$ by representing it as a linear combination of $\underline{h}_{ijl}(\mathbf{x}(t))$ and $\overline{h}_{ijl}(\mathbf{x}(t))$ in sub-FOU l as shown in (30).

Remark 6: The IT2 FMB control system in (24) is a subset of (29). Comparing both the IT2 FMB control systems, the one in (29) demonstrates some favorable properties to facilitate the stability analysis.

1) The partial information of $\underline{h}_{ijl}(\mathbf{x}(t))$ and $\overline{h}_{ijl}(\mathbf{x}(t))$ is extracted and represented by the constant scalars $\underline{\delta}_{iji_1i_2\cdots i_nkl}$ and $\overline{\delta}_{iji_1i_2\cdots i_nkl}$, which are brought to the stability conditions.
2) Referring to (25) and (26), the cross-terms $\prod_{r=1}^{n} v_{ri_rkl}(x_r(t))$ are independent of i and j and, thus, can be collected in the stability analysis.
3) With the nonlinear functions $\underline{\gamma}_{ijl}(\mathbf{x}(t))$ and $\overline{\gamma}_{ijl}(\mathbf{x}(t))$, $\tilde{h}_{ijl}(\mathbf{x}(t))$ can be reconstructed as shown in (30) as a linear combination of $\underline{h}_{ijl}(\mathbf{x}(t))$ and $\overline{h}_{ijl}(\mathbf{x}(t))$. Furthermore, with (25) and (26), the values of $\underline{h}_{ijl}(\mathbf{x}(t))$ and $\overline{h}_{ijl}(\mathbf{x}(t))$ are determined by the constant scalars $\underline{\delta}_{iji_1i_2\cdots i_nkl}$ and $\overline{\delta}_{iji_1i_2\cdots i_nkl}$ through $\prod_{r=1}^{n} v_{ri_rkl}(x_r(t))$. As a result, the stability of the IT2 FMB control system can be determined by $\underline{h}_{ijl}(\mathbf{x}(t))$ and $\overline{h}_{ijl}(\mathbf{x}(t))$ (the local lower and upper bounds of $\tilde{h}_{ij}(\mathbf{x}(t))$) characterized by the constant scalars $\underline{\delta}_{iji_1i_2\cdots i_nkl}$ and $\overline{\delta}_{iji_1i_2\cdots i_nkl}$. These properties can be seen in the stability analysis carried out in the next section.

III. Stability Analysis

The stability of the IT2 FMB control system (24) is investigated based on the Lyapunov stability theory with the consideration of the information of the lower and upper membership functions and sub-FOUs. For brevity, in the following analysis, the time t associated with the variables is dropped for the situation without ambiguity, e.g., $\mathbf{x}(t)$ is denoted as $\mathbf{x}$. The variables $\underline{w}_i(\mathbf{x}(t))$, $\overline{w}_i(\mathbf{x}(t))$, $\tilde{w}_i(\mathbf{x}(t))$, $\underline{m}_j(\mathbf{x}(t))$, $\overline{m}_j(\mathbf{x}(t))$, $\tilde{m}_j(\mathbf{x}(t))$, $\tilde{h}_{ijl}(\mathbf{x}(t))$, $v_{1i_1kl}(x_1(t))$, $v_{2i_2kl}(x_2(t)), \ldots, v_{ni_nkl}(x_n(t))$, and $\xi_{ijl}(\mathbf{x}(t))$ are denoted by $\underline{w}_i$, $\overline{w}_i$, $\tilde{w}_i$, $\underline{m}_j$, $\overline{m}_j$, $\tilde{m}_j$, $\tilde{h}_{ijl}$, v_{1i_1kl}, $v_{2i_2kl}, \ldots, v_{ni_nkl}$, and ξ_{ijl}, respectively. Furthermore, the property of $\sum_{i=1}^{p}\tilde{w}_i = \sum_{j=1}^{c}\tilde{m}_j = \sum_{i=1}^{p}\sum_{j=1}^{c}\tilde{w}_i\tilde{m}_j = \sum_{i=1}^{p}\sum_{j=1}^{c}\tilde{h}_{ij} = 1$ is utilized.

The stability analysis result is summarized in the following theorem to guarantee the asymptotic stability of the IT2 FMB control system (24) and facilitate the control synthesis.

Theorem 2: Considering the FOU being divided into $\tau+1$ sub-FOUs, the IT2 FMB control system (24) under imperfect premise matching, formed by a nonlinear plant [represented by the IT2 T–S fuzzy model (7)] and an IT2 fuzzy controller (18) connected in a closed loop, is guaranteed to be asymptotically stable if there exist matrices $\mathbf{M}=\mathbf{M}\in\Re^{n\times n}$, $\mathbf{N}_j\in\Re^{m\times n}$, $\mathbf{X}=\mathbf{X}^{\mathrm{T}}\in\Re^{n\times n}$, and $\mathbf{W}_{ijl}=\mathbf{W}_{ijl}^{\mathrm{T}}\in\Re^{n\times n}$, $i=1,2,\ldots,p$; $j=1,2,\ldots,c$; and $l=1,2,\ldots,\tau+1$, such that the following LMIs are satisfied:

$$\mathbf{X}>0 \tag{32}$$

$$\mathbf{W}_{ijl}\geq 0 \qquad \forall\, i,j,l \tag{33}$$

$$\mathbf{Q}_{ij}+\mathbf{W}_{ijl}+\mathbf{M}>0 \qquad \forall\, i,j,l \tag{34}$$

$$\sum_{i=1}^{p}\sum_{j=1}^{c}\Big(\overline{\delta}_{iji_1i_2\cdots i_nkl}\mathbf{Q}_{ij}-\big(\underline{\delta}_{iji_1i_2\cdots i_nkl}-\overline{\delta}_{iji_1i_2\cdots i_nkl}\big)\mathbf{W}_{ijl}+\overline{\delta}_{iji_1i_2\cdots i_nkl}\mathbf{M}\Big)-\mathbf{M}<0 \qquad \forall\, i_1,i_2,\ldots,i_n,k,l \tag{35}$$

where $\underline{\delta}_{iji_1i_2\cdots i_nkl}$ and $\overline{\delta}_{iji_1i_2\cdots i_nkl}$, $i=1,2,\ldots,p$; $j=1,2,\ldots,c$; $i_1,i_2,\ldots,i_n=1,2$; $k=1,2,\ldots,q$; and $l=1,2,\ldots,\tau+1$, are predefined constant scalars satisfying (25) and (26); $\mathbf{Q}_{ij}=\mathbf{A}_i\mathbf{X}+\mathbf{X}\mathbf{A}_i^{\mathrm{T}}+\mathbf{B}_i\mathbf{N}_j+\mathbf{N}_j^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}$ for all i and j; and the feedback gains are defined as $\mathbf{G}_j=\mathbf{N}_j\mathbf{X}^{-1}$ for all j.

Proof: The proof of Theorem 2 is given in the Appendix. ■

Remark 7: The stability conditions in Theorem 1 are a particular case of Theorem 2. If there exists a solution to the stability conditions in Theorem 1, $\mathbf{X}>0$ and $\mathbf{Q}_{ij}<0$ for all i and j can be achieved. Choosing $\mathbf{M}=\varepsilon_1\mathbf{I}>0$ and $\mathbf{W}_{ijl}=-\mathbf{Q}_{ij}+(-\varepsilon_1+\varepsilon_2)\mathbf{I}>0$ for all i, j, and l with sufficiently small nonzero positive values of ε_1 and ε_2 in Theorem 2, the LMIs (33) and (34) can be satisfied. As a result, recalling that $\overline{\delta}_{iji_1i_2\cdots i_nkl}\geq\underline{\delta}_{iji_1i_2\cdots i_nkl}\geq 0$, the LMIs in (35) become $\sum_{i=1}^{p}\sum_{j=1}^{c}(\overline{\delta}_{iji_1i_2\cdots i_nkl}\varepsilon_2\mathbf{I}-\underline{\delta}_{iji_1i_2\cdots i_nkl}\mathbf{W}_{ijl})-\varepsilon_1\mathbf{I}<0$ for all $i_1,i_2,\ldots,i_n$, k, and l, which will be satisfied by a sufficiently small value of ε_2. Consequently, the solution of the stability conditions in Theorem 1 is that of Theorem 2 but not the other way round.

IV. Simulation and Experimental Examples

Simulation and experimental examples are given in this section to demonstrate the effectiveness and the merit of the proposed IT2 FMB control approach.

Example 1: A three-rule IT2 T–S fuzzy model in the form of (7) is employed to represent a nonlinear plant with $\mathbf{A}_1=\begin{bmatrix}1.59 & -7.29\\ 0.01 & 0\end{bmatrix}$, $\mathbf{A}_2=\begin{bmatrix}0.02 & -4.64\\ 0.35 & 0.21\end{bmatrix}$, $\mathbf{A}_3=\begin{bmatrix}-a & -4.33\\ 0 & 0.05\end{bmatrix}$, $\mathbf{B}_1=\begin{bmatrix}1\\ 0\end{bmatrix}$, $\mathbf{B}_2=\begin{bmatrix}8\\ 0\end{bmatrix}$, $\mathbf{B}_3=\begin{bmatrix}-b+6\\ -1\end{bmatrix}$, $\mathbf{x}=[x_1\quad x_2]^{\mathrm{T}}$, and a and b being constant system parameters.

The IT2 membership functions are chosen to be $\tilde{w}_1(x_1)=\mu_{M_1^1}(x_1)=1-1/1+e^{-(x_1+4+\sigma(t))}$, $\tilde{w}_2(x_1)=\mu_{M_1^2}(x_1)=1-\tilde{w}_1(x_1)-\tilde{w}_3(x_1)$, and $\tilde{w}_3(x_1)=\mu_{M_1^3}(x_1)=1/1+e^{-(x_14+\sigma(t))}$. It should be noted that the IT2 membership functions will lead to uncertain grades of membership because of the parameter uncertainty $\sigma(t)\in[-0.1,\ 0.1]$. As a result, the existing type-1 stability analysis for FMB control system under the PDC design concept cannot be applied.

The lower and upper membership functions for the IT2 T–S fuzzy model are chosen to be $\underline{w}_1(x_1)=\underline{\mu}_{\tilde{M}_1^1}(x_1)=1-1/1+e^{-(x_1+4+d_1)}$, $\underline{w}_3(x_1)=\underline{\mu}_{\tilde{M}_1^3}(x_1)=1/1+e^{-(x_1-4-d_1)}$, $\overline{w}_1(x_1)=\overline{\mu}_{\tilde{M}_1^1}(x_1)=1-1/1+e^{-(x_1+4-d_1)}$, $\overline{w}_3(x_1)=\overline{\mu}_{\tilde{M}_1^3}(x_1)=1/1+e^{-(x_14+d_1)}$, $\underline{w}_2(x_1)=\underline{\mu}_{\tilde{M}_1^2}(x_1)=1-\overline{\mu}_{\tilde{M}_1^1}(x_1)-\overline{\mu}_{\tilde{M}_1^3}(x_1)$, and $\overline{w}_2(x_1)=\overline{\mu}_{\tilde{M}_1^2}(x_1)=1-\underline{\mu}_{\tilde{M}_1^1}(x_1)-\underline{\mu}_{\tilde{M}_1^3}(x_1)$, where d_1 is a constant to be determined.

To stabilize the nonlinear plant, a two-rule IT2 fuzzy controller in the form of (18) is employed. For demonstration purposes, the lower and upper membership functions are chosen as $\underline{m}_1(x_1)=\underline{\mu}_{\tilde{N}_1^1}(x_1)=1-1/e^{-x_1+d_2/2}$, $\overline{m}_1(x_1)=\overline{\mu}_{\tilde{N}_1^1}(x_1)=1-1/e^{-x_1-d_2/2}$, $\underline{m}_2(x_1)=\underline{\mu}_{\tilde{N}_1^2}(x_1)=1-\overline{\mu}_{\tilde{N}_1^1}(x_1)$, and $\overline{m}_2(x_1)=\overline{\mu}_{\tilde{N}_1^2}(x_1)=1-\underline{\mu}_{\tilde{N}_1^1}(x_1)$. From (19), we have $\tilde{m}_j(x_1)=(\underline{\beta}_j\underline{m}_j(x_1)+\overline{\beta}_j\overline{m}_j(x_1))\,/\,(\sum_{k=1}^{2}(\underline{\beta}_k\underline{m}_k(x_1)+\overline{\beta}_k\overline{m}_k(x_1)))$ for $j=1,2$, where $\underline{\beta}_j$ and $\overline{\beta}_j$ are chosen to be constants and d_2 is a constant to be determined.

In this example, we consider $\tau=0$, which means that no sub-FOUs are considered. For simplicity, the subscript l is dropped for all variables. To determine the (local) lower and upper membership functions $\underline{h}_{ij}(x_1)$ and $\overline{h}_{ij}(x_1)$, we consider $x_1\in[-10,\ 10]$ and divide the state space of x_1 into 20 equal-size regions (which is arbitrarily chosen for demonstration purposes), i.e., $\phi_k:\underline{x}_{1,k}\leq x_1\leq\overline{x}_{1,k}$, $k=1,2,\ldots,20$, where $\underline{x}_{1,k}=(k-11)$ and $\overline{x}_{1,k}=(k-10)$. The lower and upper membership functions $\underline{h}_{ij}(x_1)$ and $\overline{h}_{ij}(x_1)$ are defined by choosing $v_{11k}(x_1)=1-(x_1-\underline{x}_{1,k}/\underline{x}_{1,k}-\overline{x}_{1,k})$ and $v_{12k}(x_1)=1-v_{11k}(x_1)$, and the constant scalars are defined as $\underline{\delta}_{ij1k}=\underline{w}_i(\underline{x}_{1,k})\underline{m}_j(\underline{x}_{1,k})$, $\underline{\delta}_{ij2k}=\underline{w}_i(\overline{x}_{1,k})\underline{m}_j(\overline{x}_{1,k})$, $\overline{\delta}_{ij1k}=\overline{w}_i(\underline{x}_{1,k})\overline{m}_j(\underline{x}_{1,k})$, and $\overline{\delta}_{ij2k}=\overline{w}_i(\overline{x}_{1,k})\overline{m}_j(\overline{x}_{1,k})$ for all k.

It should be noted that, by employing the same lower and upper membership functions $\underline{h}_{ij}(x_1)$ and $\overline{h}_{ij}(x_1)$, any $\underline{\beta}_j$ and $\overline{\beta}_j$ in the fuzzy controller will make no difference in the stability analysis result except the implementation of the IT2 fuzzy controller. However, by employing different values of $\underline{\beta}_j$ and $\overline{\beta}_j$, the IT2 fuzzy controller defined in (18) will affect the FOU of $\tilde{h}_{ij}\equiv\tilde{w}_i(x_1)\tilde{m}_j(x_1)$. As a result, different $\underline{h}_{ij}(x_1)$ and $\overline{h}_{ij}(x_1)$ fitting better the FOU can be employed for different cases. In this example, the introduction of d_1 and d_2 to the membership functions is for the purpose of obtaining fitter $\underline{h}_{ij}(x_1)$ and $\overline{h}_{ij}(x_1)$ for different values of $\underline{\beta}_j$ and $\overline{\beta}_j$.

The stability of the IT2 FMB control system subject to different values of a and b is checked by the LMI-based stability conditions in Theorem 2 ($l=1$) with the help of the Matlab LMI toolbox. Three cases shown in Table I with different values of $\underline{\beta}_j$, $\underline{\beta}_j$, d_1, and d_2 are considered to demonstrate the characteristics of the IT2 fuzzy controller and how they influence the stabilization capability. The values of d_1 and d_2

TABLE I
PARAMETER VALUES FOR $\underline{\beta}_j$, $\overline{\beta}_j$, d_1, AND d_2 IN EXAMPLE 1

	Case 1	Case 2	Case 3
$\underline{\beta}_j$	1	0.5	0
$\overline{\beta}_j$	0	0.5	1
d_1	0.3	0.3	0.25
d_2	0.25	0.15	0.15

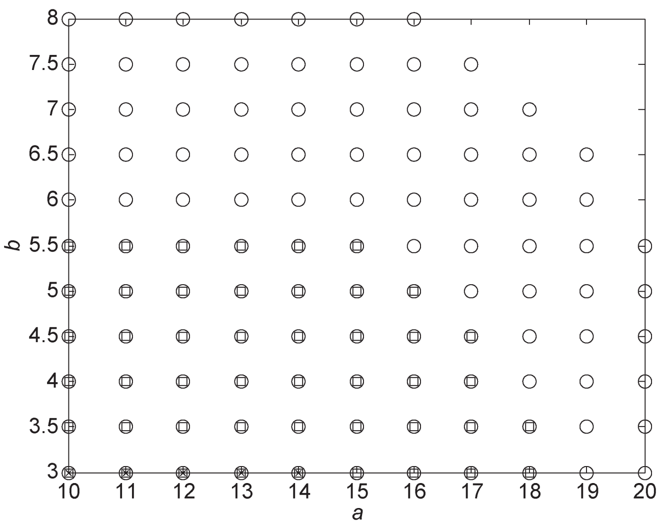

Fig. 1. Stability regions given by the stability conditions in Theorem 2 for (×) Case 1 (5 points), (□) Case 2 (41 points), and (○) Case 3 (110 points) in Example 1.

TABLE II
FEEDBACK GAINS OF THE IT2 FUZZY CONTROLLER IN EXAMPLE 2 FOR DIFFERENT VALUES OF a AND b CORRESPONDING TO THE PARAMETER VALUES OF $\underline{\beta}_j$, $\overline{\beta}_j$, d_1, AND d_2 FOR DIFFERENT CASES AS SHOWN IN TABLE I

Case	a, b	Feedback gains $\mathbf{G}_j$
1	$a = 14$, $b = 3$	$\mathbf{G}_1 = [-2.8221 \quad -2.9730]$ $\mathbf{G}_2 = [-0.4278 \quad 0.3379]$
2	$a = 15$, $b = 5.5$	$\mathbf{G}_1 = [-2.9261 \quad -3.2335]$ $\mathbf{G}_2 = [-0.3885 \quad 0.3763]$
3	$a = 20$, $b = 5.5$	$\mathbf{G}_1 = [-2.5464 \quad -2.2206]$ $\mathbf{G}_2 = [-0.6126 \quad 0.2093]$

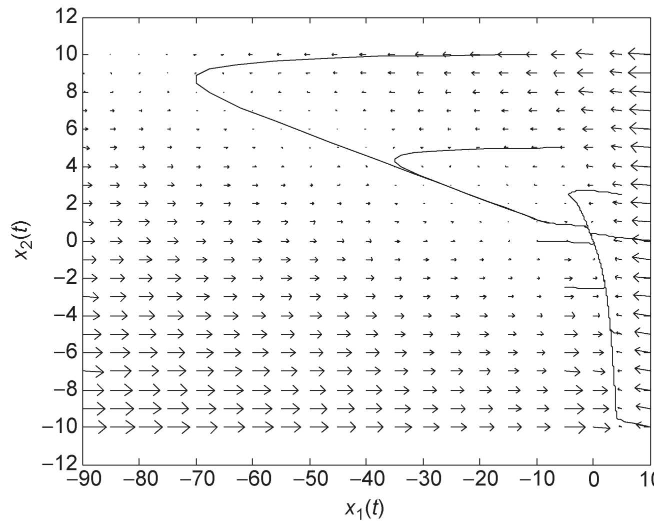

Fig. 2. Phase portrait of the system states of IT2 FMB control system subject to various initial conditions for $a = 14$ and $b = 3$, with parameter values of $\underline{\beta}_j$, $\overline{\beta}_j$, d_1, and d_2 shown in Case 1 in Table I.

are chosen such that $\tilde{h}_{ij}(x_1)$ in the form of (30) is within the lower and upper membership functions defined in (25) and (26), respectively. We consider $10 \leq a \leq 20$ at the interval of one and $3 \leq b \leq 8$ at the interval of 0.5 for each of the three cases. The stability regions corresponding to Cases 1–3 indicated by "×," "□," and "○," respectively, are shown in Fig. 1. As seen on these figures, different values of $\underline{\beta}_j$ and $\overline{\beta}_j$ leading to different values of d_1 and d_2 produce different sizes of stability regions.

For comparison purposes, Theorem 1 is employed to check the stability of the IT2 FMB control system. However, there is no feasible solution by using the Matlab LMI toolbox. It should be noted that the IT2 FMB control system is under imperfect premise matching and the stability conditions in [39] for perfect premise matching cannot be applied in this example. In order to apply the stability conditions in [39], we consider that the IT2 fuzzy controller shares the same lower and upper membership functions as those of the IT2 T–S fuzzy model. However, there is still no feasible solution for this example.

Example 2: The simulation results of the system responses for the IT2 FMB control system given in the previous example were performed for the verification of the stability analysis result. The IT2 T–S fuzzy model is given as $\dot{\mathbf{x}} = \sum_{i=1}^{3} \tilde{w}_i(x_1)(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i u)$. A two-rule IT2 fuzzy controller $u = \sum_{j=1}^{2} \tilde{m}_j(x_1)\mathbf{G}_j\mathbf{x}$ is proposed to close the feedback loop. As a result, we have the IT2 FMB control system $\dot{\mathbf{x}} = \sum_{i=1}^{3}\sum_{j=1}^{2} \tilde{w}_i(x_1)\tilde{m}_j(x_1)(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i\mathbf{G}_j)\mathbf{x}$, which can be represented in the form of (29). The membership functions are defined in the previous example. In this example, we consider that the grades of membership are capped such that $\tilde{w}_i(x_1) = \tilde{w}_i(-10)$, $i = 1, 2, 3$, and $\tilde{m}_j(x_1) = \tilde{m}_j(-10)$, $j = 1, 2$, for $x_1 \leq -10$ and $\tilde{w}_i(x_1) = \tilde{w}_i(10)$, $i = 1, 2, 3$, and $\tilde{m}_j(x_1) = \tilde{m}_j(10)$, $j = 1, 2$, for $x_1 \geq 10$ in order to apply the stability analysis result obtained in the previous example for $x_1 \in [-10,\ 10]$.

Referring to Fig. 1, we pick arbitrarily a number of points corresponding to the parameter values of $\underline{\beta}_j$, $\overline{\beta}_j$, d_1, and d_2 as shown in Table I. We consider the system parameters $a = 14$ and $b = 3$ for the parameters of Case 1 in Table I, $a = 15$ and $b = 5.5$ for Case 2, and $a = 20$ and $b = 5.5$ for Case 3 to perform the simulations. The parameter uncertainty is chosen to be $\sigma(t) = 0.1\sin(x_1) \in [-0.1,\ 0.1]$ for demonstration purposes. With the Matlab LMI toolbox and the LMI-based stability conditions in Theorem 2, we obtained the feedback gains of the IT2 fuzzy controller for different cases as shown in Table II. The phase portraits of x_1 and x_2 for different cases with various initial conditions are shown in Figs. 2–4. It can be seen that the IT2 fuzzy controllers are able to stabilize the nonlinear plant with different values of a and b.

Example 3: In this example, we investigate the effect of using the information of sub-FOUs to the size of the stability region through a computer simulation. Consider the same IT2 T–S fuzzy model and IT2 fuzzy controller as those in Example 1. The LMI-based stability conditions are employed

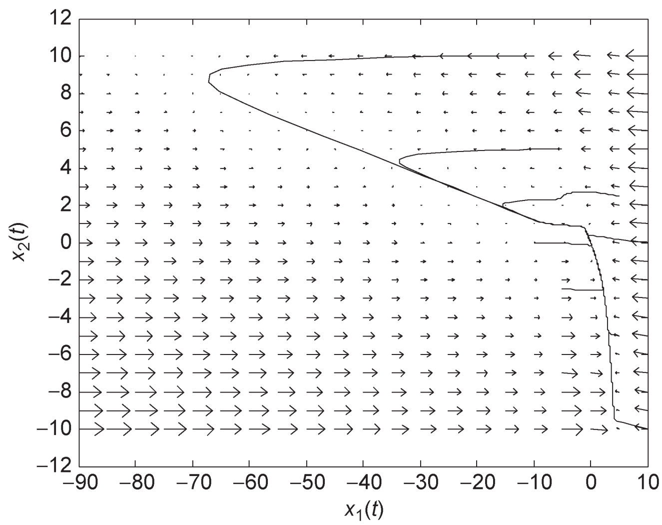

Fig. 3. Phase portrait of the system states of IT2 FMB control system subject to various initial conditions for $a = 15$ and $b = 5.5$, with parameter values of $\underline{\beta}_j$, $\underline{\beta}_j$, d_1, and d_2 shown in Case 2 in Table I.

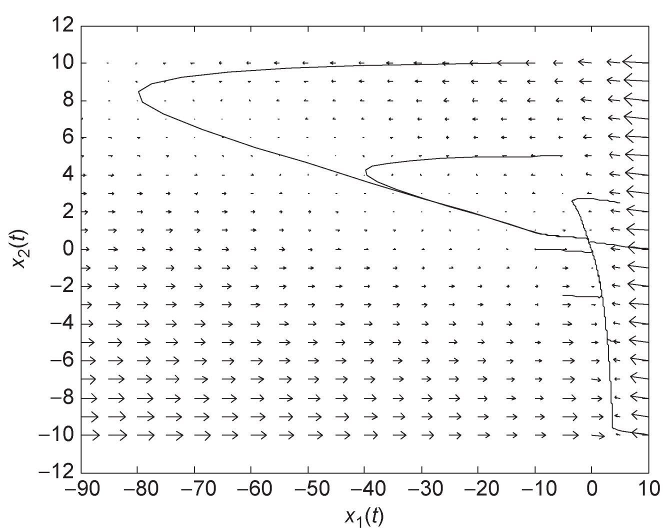

Fig. 4. Phase portrait of the system states of IT2 FMB control system subject to various initial conditions for $a = 20$ and $b = 5.5$, with parameter values of $\underline{\beta}_j$, $\underline{\beta}_j$, d_1, and d_2 shown in Case 3 in Table I.

to check the stability of the IT2 FMB control system with the system parameters $10 \leq a \leq 20$ at the interval of one and $14 \leq b \leq 50$ at the interval of two (a larger parameter range is considered compared with Example 1). Three scenarios, with different numbers of sub-FOUs from two to four, are considered and shown in Tables III–V. For each scenario, we consider the parameter values of $\underline{\beta}_j$, $\underline{\beta}_j$, d_1, and d_2 as shown in Table I. As a result, we have nine combinations in total.

The lower and upper membership functions $\underline{h}_{ij}(x_1)$ and $\overline{h}_{ij}(x_1)$ are defined in Example 1. According to Tables III–V, the local lower and upper membership functions $\underline{h}_{ijl}(x_1)$ and $\overline{h}_{ijl}(x_1)$ for sub-FOU l, $l = 1, 2, \ldots, \tau + 1$, can be defined.

With the Matlab LMI toolbox and the LMI-based stability conditions in Theorem 2, the stability regions for different scenarios and cases are shown in Figs. 5–7. Referring to these figures, it can be seen that different values of $\underline{\beta}_j$, $\underline{\beta}_j$, d_1, and d_2 will produce different sizes of stability regions. It follows the trend that Case 3 produces a larger stability region than Case 2 while Case 2 produces a larger stability region than Case 1. Comparing with Example 1, it can be seen that the stability regions shown in Figs. 5–7 are larger [it should be noted that the scale in Fig. 7 ($3 \leq b \leq 8$) is different from those in Figs. 5–7($14 \leq b \leq 50$)]. It is because more information is considered by the stability conditions in Theorem 2 through the local lower and upper membership functions $\underline{h}_{ijl}(x_1)$ and $\overline{h}_{ijl}(x_1)$. Comparing the stability regions in Figs. 5–7, it can be observed that Scenario 3 produces a larger stability region than Scenario 2 while Scenario 2 produces a larger stability region than Scenario 1 as more information is utilized when more sub-FOUs are considered.

TABLE III
LOWER AND UPPER MEMBERSHIP FUNCTIONS $\underline{h}_{ijl}(x_1)$ AND $\overline{h}_{ijl}(x_1)$, $l = 1, 2$, FOR SCENARIO 1 IN EXAMPLE 3. THE LOWER AND UPPER MEMBERSHIP FUNCTIONS $\underline{h}_{ij}(x_1)$ AND $\overline{h}_{ij}(x_1)$ ARE DEFINED IN EXAMPLE 1

τ	1
$\underline{h}_{ijl}(x_1)$	$\underline{h}_{ij1}(x_1) = \frac{\underline{h}_{ij}(x_1)+\overline{h}_{ij}(x_1)}{2}$ $\underline{h}_{ij2}(x_1) = \underline{h}_{ij}(x_1)$
$\overline{h}_{ijl}(x_1)$	$\overline{h}_{ij1}(x_1) = \overline{h}_{ij}(x_1)$ $\overline{h}_{ij2}(x_1) = \frac{\underline{h}_{ij}(x_1)+\overline{h}_{ij}(x_1)}{2}$

TABLE IV
LOWER AND UPPER MEMBERSHIP FUNCTIONS $\underline{h}_{ijl}(x_1)$ AND $\overline{h}_{ijl}(x_1)$, $l = 1, 2, 3$, FOR SCENARIO 2 IN EXAMPLE 3. THE LOWER AND UPPER MEMBERSHIP FUNCTIONS $\underline{h}_{ij}(x_1)$ AND $\overline{h}_{ij}(x_1)$ ARE DEFINED IN EXAMPLE 1

τ	2
$\underline{h}_{ijl}(x_1)$	$\underline{h}_{ij1}(x_1) = \frac{\underline{h}_{ij}(x_1)+2\overline{h}_{ij}(x_1)}{3}$ $\underline{h}_{ij2}(x_1) = \frac{2\underline{h}_{ij}(x_1)+\overline{h}_{ij}(x_1)}{3}$ $\underline{h}_{ij3}(x_1) = \underline{h}_{ij}(x_1)$
$\overline{h}_{ijl}(x_1)$	$\overline{h}_{ij1}(x_1) = \overline{h}_{ij}(x_1)$ $\overline{h}_{ij2}(x_1) = \frac{\underline{h}_{ij}(x_1)+2\overline{h}_{ij}(x_1)}{3}$ $\overline{h}_{ij3}(x_1) = \frac{2\underline{h}_{ij}(x_1)+\overline{h}_{ij}(x_1)}{3}$

TABLE V
LOWER AND UPPER MEMBERSHIP FUNCTIONS $\underline{h}_{ijl}(x_1)$ AND $\overline{h}_{ijl}(x_1)$, $l = 1, 2, 3, 4$, FOR SCENARIO 3 IN EXAMPLE 3. THE LOWER AND UPPER MEMBERSHIP FUNCTIONS $\underline{h}_{ij}(x_1)$ AND $\overline{h}_{ij}(x_1)$ ARE DEFINED IN EXAMPLE 1

τ	3
$\underline{h}_{ijl}(x_1)$	$\underline{h}_{ij1}(x_1) = \frac{\underline{h}_{ij}(x_1)+3\overline{h}_{ij}(x_1)}{4}$ $\underline{h}_{ij2}(x_1) = \frac{\underline{h}_{ij}(x_1)+\overline{h}_{ij}(x_1)}{2}$ $\underline{h}_{ij3}(x_1) = \frac{3\underline{h}_{ij}(x_1)+\overline{h}_{ij}(x_1)}{4}$ $\underline{h}_{ij4}(x_1) = \underline{h}_{ij}(x_1)$
$\overline{h}_{ijl}(x_1)$	$\overline{h}_{ij1}(x_1) = \overline{h}_{ij}(x_1)$ $\overline{h}_{ij2}(x_1) = \frac{\underline{h}_{ij}(x_1)+3\overline{h}_{ij}(x_1)}{4}$ $\overline{h}_{ij3}(x_1) = \frac{\underline{h}_{ij}(x_1)+\overline{h}_{ij}(x_1)}{2}$ $\overline{h}_{ij4}(x_1) = \frac{3\underline{h}_{ij}(x_1)+\overline{h}_{ij}(x_1)}{4}$

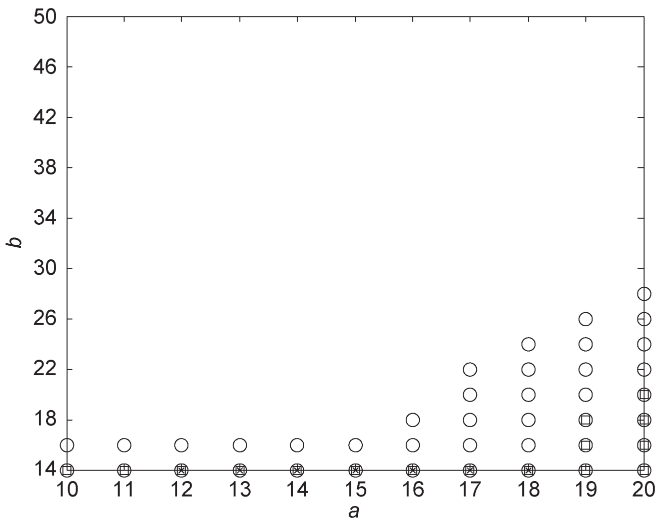

Fig. 5. Stability regions of Scenario 1 (lower and upper membership functions defined in Table III) given by the stability conditions in Theorem 2 for (×) Case 1 (7 points), (□) Case 2 (16 points), and (○) Case 3 (41 points) in Example 3.

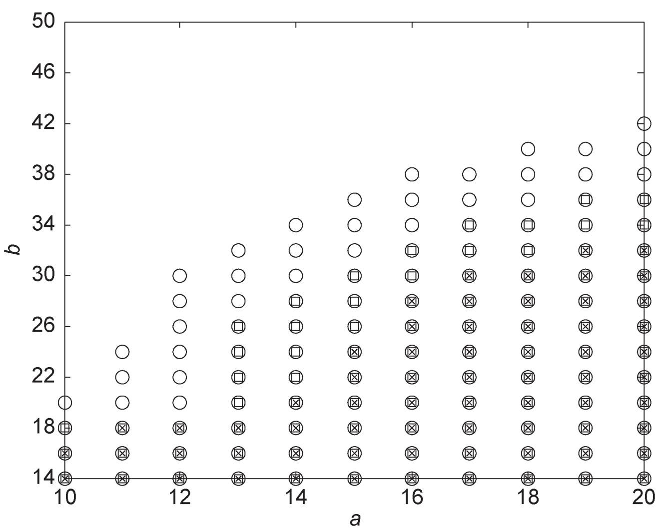

Fig. 6. Stability regions of Scenario 2 (lower and upper membership functions defined in Table IV) given by the stability conditions in Theorem 2 for (×) Case 1 (67 points), (□) Case 2 (89 points), and (○) Case 3 (121 points) in Example 3.

Example 4: In this example, we consider an inverted pendulum, as shown in Fig. 8, subject to parameter uncertainties [39] as the nonlinear plant to be controlled. The dynamic equation for the inverted pendulum is given by

$$\ddot{\theta}(t)=\frac{g\sin(\theta(t))-am_pL\dot{\theta}(t)^2\sin(2\theta(t))/2-a\cos(\theta(t))u(t)}{4L/3-am_pL\cos^2(\theta(t))} \tag{36}$$

where $\theta(t)$ is the angular displacement of the pendulum, $g=9.8$ m/s^2 is the acceleration due to gravity, $m_p\in[m_{p_{\min}}\quad m_{p_{\max}}]=[2\quad 3]$ kg is the mass of the pendulum, $M_c\in[M_{\min}\quad M_{\max}]=[8\quad 12]$ kg is the mass of the cart, $a=1/(m_p+M_c)$, $2L=1$ m is the length of the pendulum, and $u(t)$ is the force (in newtons) applied to the cart. The inverted pendulum is considered working in the operating

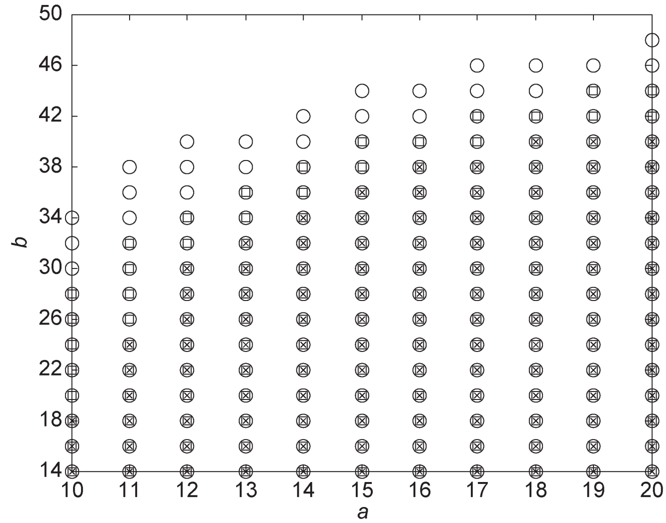

Fig. 7. Stability regions of Scenario 3 (lower and upper membership functions defined in Table V) given by the stability conditions in Theorem 2 for (×) Case 1 (125 points), (□) Case 2 (144 points), and (○) Case 3 (168 points) in Example 3.

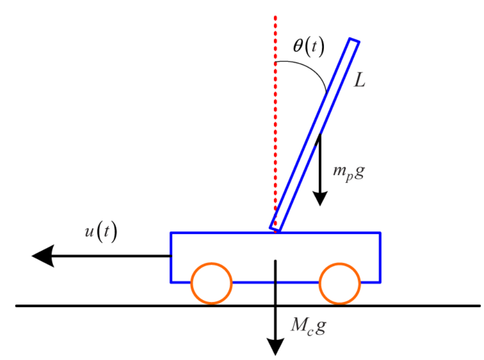

Fig. 8. Inverted pendulum system.

domain characterized by $x_1=\theta(t)\in[-5\pi/12,\quad 5\pi/12]$ and $x_2=\dot{\theta}(t)\in[-5,\quad 5]$.

A four-rule IT2 T–S fuzzy model in the form of (7) is employed to describe the inverted pendulum subject to parameter uncertainties with $\mathbf{x}=[x_1\quad x_2]^{\mathrm{T}}=[\theta(t)\quad \dot{\theta}(t)]^{\mathrm{T}}$, $\mathbf{A}_1=\mathbf{A}_2=\begin{bmatrix}0 & 1\\ f_{1_{\min}} & 0\end{bmatrix}$, $\mathbf{A}_3=\mathbf{A}_4=\begin{bmatrix}0 & 1\\ f_{1_{\max}} & 0\end{bmatrix}$, $\mathbf{B}_1=\mathbf{B}_3=\begin{bmatrix}0\\ f_{2_{\min}}\end{bmatrix}$, $\mathbf{B}_2=\mathbf{B}_4=\begin{bmatrix}0\\ f_{2_{\max}}\end{bmatrix}$, $f_{1_{\min}}=10.0078$, $f_{1_{\max}}=18.4800$, $f_{2_{\min}}=-0.1765$, and $f_{2_{\max}}=-0.0261$. The lower and upper membership functions are defined in Table VI.

A two-rule IT2 fuzzy controller is employed to stabilize the inverted pendulum with the lower and upper membership functions chosen as $\underline{m}_1(x_1)=\underline{\mu}_{\tilde{N}_1^1}(x_1)=\overline{m}_1(x_1)=\overline{\mu}_{\tilde{N}_1^1}(x_1)=e^{-x_1^2/0.35}$, $\underline{m}_2(x_1)=\underline{\mu}_{\tilde{N}_1^2}(x_1)=\overline{m}_2(x_1)=\overline{\mu}_{\tilde{N}_1^2}(x_1)=1-\overline{\mu}_{\tilde{N}_1^1}(x_1)$, and $\underline{\beta}_k=\overline{\beta}_k=1/2$.

In this example, we consider only one sub-FOU, i.e., $\tau=0$. For simplicity, the subscript l is dropped for all variables. The number of equal-size regions for x_1 is arbitrarily chosen to be 500. The lower and upper membership functions $\underline{h}_{ij}(x_1)$ and $\overline{h}_{ij}(x_1)$ are defined by choosing $v_{11k}(x_1)=1-(x_1-\underline{x}_{1,k}/\underline{x}_{1,k}-\overline{x}_{1,k})$ and $v_{12k}(x_1)=1-v_{11k}(x_1)$,

TABLE VI
LOWER AND UPPER MEMBERSHIP FUNCTIONS OF THE IT2 T–S FUZZY MODEL OF INVERTED PENDULUM IN EXAMPLE 4

Lower membership functions	Upper membership functions
$\underline{\mu}_{\tilde{M}_1^1}(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$\overline{\mu}_{\tilde{M}_1^1}(x_1) = 1 - 0.23e^{-\frac{x_1^2}{0.25}}$
$\underline{\mu}_{\tilde{M}_1^2}(x_1) = 1 - e^{-\frac{x_1^2}{1.2}}$	$\overline{\mu}_{\tilde{M}_1^2}(x_1) = 1 - 0.23e^{-\frac{x_1^2}{0.25}}$
$\underline{\mu}_{\tilde{M}_1^3}(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$\overline{\mu}_{\tilde{M}_1^3}(x_1) = e^{-\frac{x_1^2}{1.2}}$
$\underline{\mu}_{\tilde{M}_1^4}(x_1) = 0.23e^{-\frac{x_1^2}{0.25}}$	$\overline{\mu}_{\tilde{M}_1^4}(x_1) = e^{-\frac{x_1^2}{1.2}}$
$\underline{\mu}_{\tilde{M}_2^1}(x_1) = 0.5e^{-\frac{x_1^2}{0.25}}$	$\overline{\mu}_{\tilde{M}_2^1}(x_1) = e^{-\frac{x_1^2}{1.5}}$
$\underline{\mu}_{\tilde{M}_2^2}(x_1) = 1 - e^{-\frac{x_1^2}{1.5}}$	$\overline{\mu}_{\tilde{M}_2^2}(x_1) = 1 - 0.5e^{-\frac{x_1^2}{0.25}}$
$\underline{\mu}_{\tilde{M}_2^3}(x_1) = 0.5e^{-\frac{x_1^2}{0.25}}$	$\overline{\mu}_{\tilde{M}_2^3}(x_1) = e^{-\frac{x_1^2}{1.5}}$
$\underline{\mu}_{\tilde{M}_2^4}(x_1) = 1 - e^{-\frac{x_1^2}{1.5}}$	$\overline{\mu}_{\tilde{M}_2^4}(x_1) = 1 - 0.5e^{-\frac{x_1^2}{0.25}}$

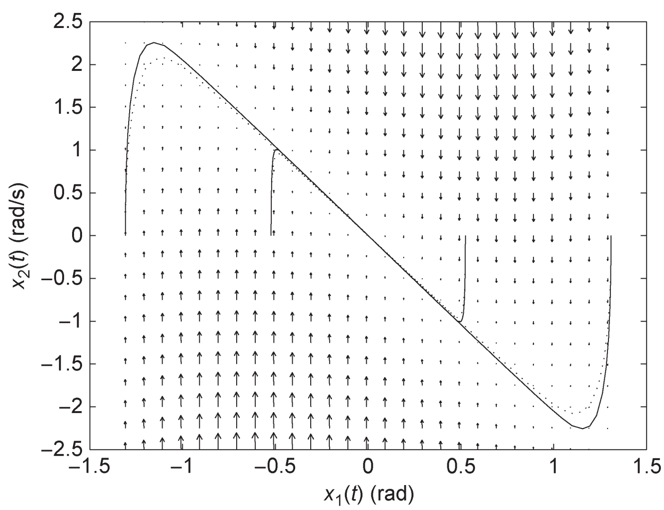

Fig. 9. Phase portrait of the system states of the inverted pendulum subject to various initial conditions. (Solid lines) $m_p = 2$ kg and $M_c = 8$ kg. (Dotted lines) $m_p = 3$ kg and $M_c = 12$ kg.

where $\underline{x}_{1,k} = (10\pi/12)/500(k - 251)$ and $\overline{x}_{1,k} = (10\pi/12)/500(k - 250)$, $k = 1, 2, \ldots, 500$. The constant scalars are chosen as $\underline{\delta}_{ij1k} = \underline{w}_i(\underline{x}_{1,k})\underline{m}_j(\underline{x}_{1,k})$, $\underline{\delta}_{ij2k} = \underline{w}_i(\overline{x}_{1,k})\underline{m}_j(\overline{x}_{1,k})$, $\overline{\delta}_{ij1k} = \overline{w}_i(\underline{x}_{1,k})\overline{m}_j(\underline{x}_{1,k})$, and $\overline{\delta}_{ij2k} = \overline{w}_i(\overline{x}_{1,k})\overline{m}_j(\overline{x}_{1,k})$ for all k.

Theorem 2 with $l = 1$ is employed to determine the system stability and synthesize the feedback gains. A feasible solution was found as $\mathbf{X} = \begin{bmatrix} 0.0983 & -0.1870 \\ -0.1870 & 0.4989 \end{bmatrix}$, $\mathbf{G}_1 = [1432.8239 \quad 653.0531]$, and $\mathbf{G}_2 = [1845.9736 \quad 849.8562]$. The IT2 fuzzy controller is employed to stabilize the inverted pendulum with $m_p = 2$ kg and $M_c = 8$ kg and with $m_p = 3$ kg and $M_c = 12$ kg, respectively. The phase portrait of the system states is shown in Fig. 9, which shows that the inverted pendulum can be stabilized subject to different values of m_p and M_c and different initial conditions.

For comparison purposes, considering the simulation result in [39], it can be seen that the IT2 fuzzy controller can also stabilize the inverted pendulum. However, the number of rules of the IT2 fuzzy controller is required to be four because of the PDC design concept. In this example, the IT2 T–S fuzzy model and fuzzy controller do not share the same premise membership functions and the same number of rules. Consequently, the stability conditions proposed in [39] cannot be applied in this example. Furthermore, because the number of rules is two and simpler membership functions are used, the implementation complexity of the IT2 fuzzy controller is reduced.

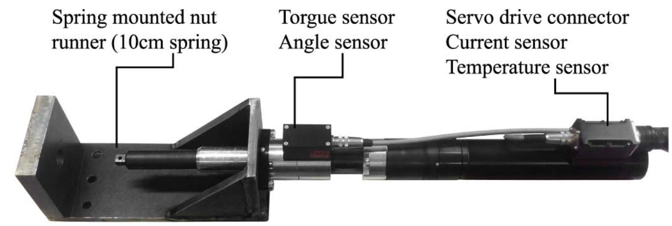

Fig. 10. Bolt-tightening tool.

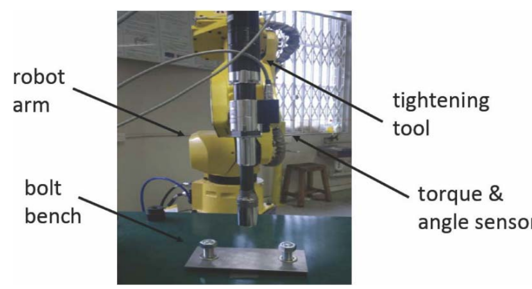

Fig. 11. Bolt-tightening tool mounted on a robot arm.

Example 5: An experiment was done to verify the analysis result. A bolt-tightening tool (DSM BL 57/140 MDW), which is shown in Fig. 10, is considered as the plant. In real operation, the bolt-tightening tool is mounted on a robot arm (Fanuc M6iB) for bolt tightening, as shown in Fig. 11. An integrated encoder and a torque sensor are installed to provide the information of angular position (360° per revolution) and torque, respectively. It accepts voltage in the range of -10 to 10 V as input.

An IT2 fuzzy model is constructed to describe the system dynamics with the Matlab system identification toolbox. A local state-space model was obtained using the input–output data, which are the input voltage and the output angle position and angular velocity. Three local state-space models operating at output angles around $-90°$, 0, and 90° were obtained under no-load condition. IT2 fuzzy sets are employed to combine the three local state-space models to form an IT2 fuzzy model to facilitate the design of the IT2 fuzzy controller. The IT2 fuzzy model was obtained in the form of (7) with $\mathbf{x} = [x_1 \quad x_2]^{\mathrm{T}}$, where x_1 is the angle position in degrees and x_2 is the angular velocity in degrees per second, $\mathbf{A}_1 = \begin{bmatrix} 0.0009 & 0.0034 \\ 0.0108 & -0.0264 \end{bmatrix}$, $\mathbf{A}_2 = \begin{bmatrix} 0.0008 & 0.0042 \\ 0.098 & -0.0161 \end{bmatrix}$, $\mathbf{A}_3 = \begin{bmatrix} 0.0008 & 0.0050 \\ 0.088 & -0.0057 \end{bmatrix}$, $\mathbf{B}_1 = \begin{bmatrix} 0.0014 \\ 0.0013 \end{bmatrix}$, $\mathbf{B}_2 = \begin{bmatrix} 0.0014 \\ 0.0016 \end{bmatrix}$, and $\mathbf{B}_3 = \begin{bmatrix} 0.0014 \\ 0.0018 \end{bmatrix}$. The lower and upper membership functions are chosen as

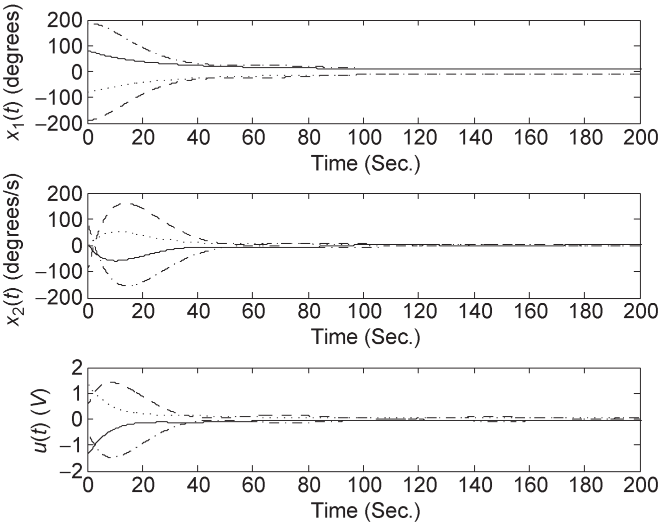

Fig. 12. State responses and control signals of the IT2-FMB-controlled bolt-tightening tool subject to initial conditions of (dotted line) $\mathbf{x}(0) = [-180 \quad 0]^{\mathrm{T}}$, (dotted lines) $\mathbf{x}(0) = [-75 \quad 0]^{\mathrm{T}}$, (solid lines) $\mathbf{x}(0) = [75 \quad 0]^{\mathrm{T}}$, and (dash-dotted lines) $\mathbf{x}(0) = [180 \quad 0]^{\mathrm{T}}$.

$\underline{w}_1(x_1) = \underline{\mu}_{\tilde{M}_1^1}(x_1) = 0.8 - (0.8/1 + e^{-x_1+90/15})$, $\underline{w}_3(x_1) = \underline{\mu}_{\tilde{M}_1^3}(x_1) = 0.8/1 + e^{-x_1-90/15}$, $\overline{w}_1(x_1) = \overline{\mu}_{\tilde{M}_1^1}(x_1) = 1 - 1/1 + e^{-x_1+90/15}$, $\overline{w}_3(x_1) = \overline{\mu}_{\tilde{M}_1^3}(x_1) = 1/1 + e^{-x_1-90/15}$, $\underline{w}_2(x_1) = \underline{\mu}_{\tilde{M}_1^2}(x_1) = 1 - \overline{\mu}_{\tilde{M}_1^1}(x_1) - \overline{\mu}_{\tilde{M}_1^3}(x_1)$, and $\overline{w}_2(x_1) = \overline{\mu}_{\tilde{M}_1^2}(x_1) = 1 - \underline{\mu}_{\tilde{M}_1^1}(x_1) - \underline{\mu}_{\tilde{M}_1^3}(x_1)$.

A two-rule IT2 fuzzy controller is employed to stabilize the angle position, where the lower and upper membership functions are chosen as $\underline{m}_1(x_1) = \underline{\mu}_{\tilde{N}_1^1}(x_1) = \overline{m}_1(x_1) = \overline{\mu}_{\tilde{N}_1^1}(x_1) = e^{-x_1^2/4000}$, $\underline{m}_2(x_1) = \underline{\mu}_{\tilde{N}_1^2}(x_1) = \overline{m}_2(x_1) = \overline{\mu}_{\tilde{N}_1^2}(x_1) = 1 - \overline{\mu}_{\tilde{N}_1^1}(x_1)$, and $\underline{\beta}_k = \overline{\beta}_k = 1/2$.

Similar to the previous example, we consider only one sub-FOU, i.e., $\tau = 0$, and thus, the subscript l is dropped for all variables. The number of equal-size regions for x_1 is arbitrarily chosen to be 500. The lower and upper membership functions $\underline{h}_{ij}(x_1)$ and $\overline{h}_{ij}(x_1)$ are defined by choosing $v_{11k}(x_1) = 1 - (x_1 - \underline{x}_{1,k}/\underline{x}_{1,k} - \overline{x}_{1,k})$ and $v_{12k}(x_1) = 1 - v_{11k}(x_1)$, where $\underline{x}_{1,k} = (10\pi/12)/500(k - 251)$ and $\overline{x}_{1,k} = (10\pi/12)/500(k - 250)$, $k = 1, 2, \ldots, 500$. The constant scalars are chosen as $\underline{\delta}_{ij1k} = \underline{w}_i(\underline{x}_{1,k})\underline{m}_j(\underline{x}_{1,k})$, $\underline{\delta}_{ij2k} = \underline{w}_i(\overline{x}_{1,k})\underline{m}_j(\overline{x}_{1,k})$, $\overline{\delta}_{ij1k} = \overline{w}_i(\underline{x}_{1,k})\overline{m}_j(\underline{x}_{1,k})$, and $\overline{\delta}_{ij2k} = \overline{w}_i(\overline{x}_{1,k})\overline{m}_j(\overline{x}_{1,k})$ for all k.

A feasible solution to Theorem 2 with $l = 1$ was found as $\mathbf{X} = \begin{bmatrix} 0.3917 & -1.3310 \\ -1.3310 & 4.6466 \end{bmatrix} \times 10^8$, $\mathbf{G}_1 = [-15.5289 \quad -4.6345]$, and $\mathbf{G}_2 = [-3.9267 \quad -1.1719]$. The IT2 fuzzy controller was implemented with a programmable logic controller which integrates Matlab Simulink in real time in a Beckhoff TwinCAT 3 system. The state responses and control signals of the IT2 FMB control system subject to initial conditions of $\mathbf{x}(0) = [-180 \quad 0]^{\mathrm{T}}$, $[-75 \quad 0]^{\mathrm{T}}$, $[75 \quad 0]^{\mathrm{T}}$, and $[180 \quad 0]^{\mathrm{T}}$ are shown in Fig. 12. The system states and control signals were sampled at 0.05 s and filtered by a tenth-order lower pass filter at the signal collection points. It can be seen from the figures that the IT2 fuzzy controller is able to stabilize the angle position, however, with a small steady error, which is due to the friction of the gearbox.

V. Conclusion

The stability of IT2 FMB control systems subject to parameter uncertainties has been investigated. Under the imperfect premise matching, the IT2 fuzzy controller can choose freely the premise membership functions and the number of rules different from those of the IT2 T–S fuzzy model, enhancing the design flexibility and reducing the implementation complexity. To facilitate the stability analysis, a favorable form of lower and upper membership functions has been proposed, and the information of sub-FOUs has been considered. The information of membership functions has been brought to the LMI-based stability conditions, resulting in a more relaxed stability analysis result. Simulation and experimental results have been given to illustrate the merit of the proposed approach. In future work, we will consider the problems of output-feedback control and sampled-data control for the nonlinear systems subject to parameter uncertainties in the frame of this paper.

Appendix
Proof of Theorem 2

We consider the following quadratic Lyapunov function candidate to investigate the stability of the IT2 FMB control systems (24) expressed in the form of (29):

$$V = \mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{x} \tag{37}$$

where $0 < \mathbf{P} = \mathbf{P}^{\mathrm{T}} \in \Re^{n \times n}$.

The main objective is to develop a condition guaranteeing that $V > 0$ and $\dot{V} < 0$ for all $\mathbf{x} \neq \mathbf{0}$. According to the Lyapunov stability theorem, by satisfying $V > 0$ and $\dot{V} < 0$ for all $\mathbf{x} \neq \mathbf{0}$, the IT2 FMB control system is guaranteed to be asymptotically stable, implying that $\mathbf{x} \to \mathbf{0}$ as time $t \to \infty$.

Denote $\mathbf{z} = \mathbf{X}^{-1}\mathbf{x}$ and $\mathbf{X} = \mathbf{P}^{-1}$. Define the feedback gains $\mathbf{G}_j = \mathbf{N}_j\mathbf{X}^{-1}$, where $\mathbf{N}_j \in \Re^{m \times n}$, $j = 1, 2, \ldots, c$, represents the matrices to be determined. From (29) and (37), we have

$$\begin{aligned} \dot{V} &= \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{P}\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathbf{P}\dot{\mathbf{x}} \\ &= \sum_{i=1}^{p}\sum_{j=1}^{c} \tilde{h}_{ij}\mathbf{x}^{\mathrm{T}}\left((\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j)^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j)\right)\mathbf{x} \\ &= \sum_{i=1}^{p}\sum_{j=1}^{c} \tilde{h}_{ij}\mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{P}^{-1}\left((\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j)^{\mathrm{T}}\mathbf{P} \right. \\ &\qquad \left. + \mathbf{P}(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j)\right)\mathbf{P}^{-1}\mathbf{P}\mathbf{x} \\ &= \sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{l=1}^{\tau+1} \xi_{ijl}\left(\underline{\gamma}_{ijl}\underline{h}_{ijl} + \overline{\gamma}_{ijl}\overline{h}_{ijl}\right)\mathbf{z}^{\mathrm{T}}\mathbf{Q}_{ij}\mathbf{z} \end{aligned} \tag{38}$$

where $\mathbf{Q}_{ij} = \mathbf{A}_i\mathbf{X} + \mathbf{X}\mathbf{A}_i^{\mathrm{T}} + \mathbf{B}_i\mathbf{N}_j + \mathbf{N}_j^{\mathrm{T}}\mathbf{B}_i^{\mathrm{T}}$.

Recalling the properties that $0 \leq \underline{h}_{ijl} \leq \overline{h}_{ijl} \leq 1$, $0 \leq \underline{\gamma}_{ijl} \leq 1$, $0 \leq \overline{\gamma}_{ijl} \leq 1$, and $\underline{\gamma}_{ijl} + \overline{\gamma}_{ijl} = 1$ for all i, j, and l, the information of the sub-FOUs is brought to the stability analysis with the introduction of some slack matrices through the following inequalities using the S procedure [14]:

$$\left(\sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{l=1}^{\tau+1}\xi_{ijl}\left(\underline{\gamma}_{ijl}\underline{h}_{ijl}+\overline{\gamma}_{ijl}\overline{h}_{ijl}\right)-1\right)\mathbf{M}=\mathbf{0} \quad (39)$$

$$-\sum_{i=1}^{p}\sum_{j=1}^{c}\left(1-\underline{\gamma}_{ijl}\right)\left(\underline{h}_{ijl}-\overline{h}_{ijl}\right)\mathbf{W}_{ijl}\geq 0 \quad (40)$$

where $\mathbf{M}=\mathbf{M}^{\mathrm{T}}\in\Re^{n\times n}$ represents arbitrary matrices and $0\leq\mathbf{W}_{ijl}=\mathbf{W}_{ijl}^{\mathrm{T}}\in\Re^{n\times n}$.

From (30), (38), (39), and (40), we have

$$\begin{aligned}\dot{V}&=\sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{l=1}^{\tau+1}\xi_{ijl}\left(\underline{\gamma}_{ijl}\underline{h}_{ijl}+\overline{\gamma}_{ijl}\overline{h}_{ijl}\right)\mathbf{z}^{\mathrm{T}}\mathbf{Q}_{ij}\mathbf{z}\\
&\leq\sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{l=1}^{\tau+1}\xi_{ijl}\left(\underline{\gamma}_{ijl}\underline{h}_{ijl}+\left(1-\underline{\gamma}_{ijl}\right)\overline{h}_{ijl}\right)\mathbf{z}^{\mathrm{T}}\mathbf{Q}_{ij}\mathbf{z}\\
&\quad-\sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{l=1}^{\tau+1}\xi_{ijl}\left(1-\underline{\gamma}_{ijl}\right)\left(\underline{h}_{ijl}-\overline{h}_{ijl}\right)\mathbf{z}^{\mathrm{T}}\mathbf{W}_{ijl}\mathbf{z}\\
&\quad+\left(\sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{l=1}^{\tau+1}\xi_{ijl}\left(\underline{\gamma}_{ijl}\underline{h}_{ijl}\right.\right.\\
&\quad\left.\left.+\left(1-\underline{\gamma}_{ijl}\right)\overline{h}_{ijl}\right)-1\right)\mathbf{z}^{\mathrm{T}}\mathbf{M}\mathbf{z}\\
&=\mathbf{z}^{\mathrm{T}}\left(\sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{l=1}^{\tau+1}\xi_{ijl}\left(\overline{h}_{ijl}\mathbf{Q}_{ij}\right.\right.\\
&\quad\left.\left.-\left(\underline{h}_{ijl}-\overline{h}_{ijl}\right)\mathbf{W}_{ijl}+\overline{h}_{ijl}\mathbf{M}\right)-\mathbf{M}\right)\mathbf{z}\\
&\quad+\sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{l=1}^{\tau+1}\xi_{ijl}\underline{\gamma}_{ijl}\left(\underline{h}_{ijl}-\overline{h}_{ijl}\right)\mathbf{z}^{\mathrm{T}}\\
&\quad\times(\mathbf{Q}_{ij}+\mathbf{W}_{ijl}+\mathbf{M})\mathbf{z}. \end{aligned} \quad (41)$$

Referring to (41), $\dot{V}<0$ for $\mathbf{x}\neq\mathbf{0}$ is satisfied by $\sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{l=1}^{\tau+1}\xi_{ijl}(\mathbf{x})(\overline{h}_{ijl}\mathbf{Q}_{ij}-(\underline{h}_{ijl}-\overline{h}_{ijl})\mathbf{W}_{ijl}+\overline{h}_{ijl}\mathbf{M})-\mathbf{M}<0$ and $\mathbf{Q}_{ij}+\mathbf{W}_{ijl}+\mathbf{M}>0$ (because of $\underline{h}_{ijl}-\overline{h}_{ijl}\leq 0$) for all i, j, and l. Recalling that only one $\xi_{ijl}=1$ for each fixed value of ij at any time instant such that $\sum_{l=1}^{\tau+1}\xi_{ijl}=1$, the first set of inequalities is satisfied by $\sum_{i=1}^{p}\sum_{j=1}^{c}(\overline{h}_{ijl}\mathbf{Q}_{ij}-(\underline{h}_{ijl}-\overline{h}_{ijl})\mathbf{W}_{ijl}+\overline{h}_{ijl}\mathbf{M})-\mathbf{M}<0$ for all i, j, and l. Expressing $\underline{h}_{ijl}$ and $\overline{h}_{ijl}$ with (25) and (26), respectively, and recalling that $\sum_{k=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{r=1}^{n}v_{ri_rkl}=1$ for all l and $v_{ri_rkl}\geq 0$ for all r, i_r, k, and l, the first set of inequalities will be satisfied if the following inequalities hold:

$$\begin{aligned}&\sum_{k=1}^{q}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{r=1}^{n}v_{ri^rkl}\\
&\times\left(\sum_{i=1}^{p}\sum_{j=1}^{c}\left(\overline{\delta}_{iji_1i_2\cdots i_nkl}\mathbf{Q}_{ij}\right.\right.\\
&\quad-\left(\underline{\delta}_{iji_1i_2\cdots i_nkl}-\overline{\delta}_{iji_1i_2\cdots i_nkl}\right)\mathbf{W}_{ijl}\\
&\quad\left.\left.+\overline{\delta}_{iji_1i_2\cdots i_nkl}\mathbf{M}\right)-\mathbf{M}\right)\\
&<0\qquad\forall\, i_1,i_2,\cdots,i_n,k,l.\end{aligned} \quad (42)$$

Consequently, $\sum_{i=1}^{p}\sum_{j=1}^{c}(\overline{h}_{ijl}\mathbf{Q}_{ij}-(\underline{h}_{ijl}-\overline{h}_{ijl})\times\mathbf{W}_{ijl}+\overline{h}_{ijl}\mathbf{M})-\mathbf{M}<0$ can be guaranteed by $\sum_{i=1}^{p}\sum_{j=1}^{c}(\overline{\delta}_{iji_1i_2\cdots i_nkl}\mathbf{Q}_{ij}-(\underline{\delta}_{iji_1i_2\cdots i_nkl}-\overline{\delta}_{iji_1i_2\cdots i_nkl})\times\mathbf{W}_{ijl}+\overline{\delta}_{iji_1i_2\cdots i_nkl}\mathbf{M})-\mathbf{M}<0$.

The LMI-based stability conditions previously mentioned are summarized in Theorem 2. By satisfying those LMIs, the IT2 FMB control system (24) is guaranteed to be asymptotically stable.

Referring to (42), the advantages of representing the IT2 FMB control system (24) in the form of (29) can be seen. The membership functions $\tilde{h}_{ij}$ are reconstructed by the linear combination of the local lower and upper membership functions $\underline{h}_{ijl}$ and $\overline{h}_{ijl}$. Consequently, as seen from (41), the stability of the IT2 FMB control system is determined by the local lower and upper membership functions $\underline{h}_{ijl}$ and $\overline{h}_{ijl}$. By expressing $\underline{h}_{ijl}$ and $\overline{h}_{ijl}$ in the form of (25) and (26), respectively, they are characterized by the constant scalars $\underline{\delta}_{iji_1i_2\cdots i_nkl}$ and $\overline{\delta}_{iji_1i_2\cdots i_nkl}$. Furthermore, as the cross-terms $\prod_{r=1}^{n}v_{ri_rkl}$ are independent of i and j, they can be extracted as shown in (42) to facilitate the stability analysis. With these favorable properties as previously stated in Remark 6, we only need to check $\sum_{i=1}^{p}\sum_{j=1}^{c}(\overline{\delta}_{iji_1i_2\cdots i_nkl}\mathbf{Q}_{ijl}-(\underline{\delta}_{iji_1i_2\cdots i_nkl}-\overline{\delta}_{iji_1i_2\cdots i_nkl})\mathbf{W}_{ijl}+\overline{\delta}_{iji_1i_2\cdots i_nkl}\mathbf{M})-\mathbf{M}<0$ at some discrete points ($\underline{\delta}_{iji_1i_2\cdots i_nkl}$ and $\overline{\delta}_{iji_1i_2\cdots i_nkl}$) instead of every single point of the local lower and upper membership functions $\underline{h}_{ijl}$ and $\overline{h}_{ijl}$ to guarantee the holding of the inequality (42).

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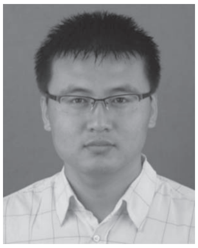

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