Planar Shape Changing Compliant Tensegrity Mechanisms with Multi-Stable Equilibria

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***Abstract*—Compliant Tensegrity Mechanisms (CoTM) combine springs, cables and rigid rods for making robust, shape-changing mechanical structures where the equilibrium equations are non-linear in the orientation $(\sin\theta, \cos\theta)$. Moreover, the mechanism orientation may be controlled through its compliance, such as, by indirectly varying free lengths of the springs and also through the length of the compressive rigid member. Interestingly, the non-linearities allow the mechanism to exist in multiple equilibrium positions. The research presents a case of a planar CoTM that exists in two stable and three unstable equilibrium positions.**

***Index Terms*—Tensegrity, compliant, mechanisms, multiple equilibria, shape changing structures.**

I. Planar Compliant Tensegrity Mechanism

The proposed mechanism consists of two triangular rigid bodies connected by a rigid rod and two springs members as shown in Fig. 1 where the relative distance between points $1, 4$ is L_3. The spring free lengths and spring constants are denoted by L_{0i}, k_i where $i = 1, 2$. where f_3 is the unknown

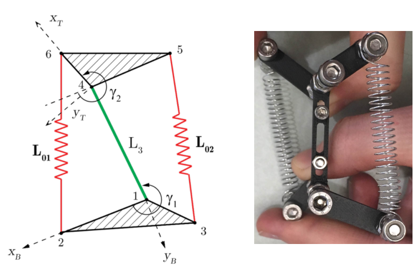

Fig. 1: The proposed planar Compliant Tensegrity Mechanism (CoTM) comprises of two triangular rigid bodies connected by a rigid rod (green) and two springs (red) alongside the physical model

force along the rigid bar, $f_i = k_i(d_i - L_{0i})$ are the forces in spring elements and d_i are length of the spring elements $\forall i = 1, 2$. The four equations of interest are two equilibrium equations coupled with two constraint equations

$$k_1\left(1 - \frac{L_{01}}{d_1}\right)(\boldsymbol{P}_{1\to4} \times \boldsymbol{P}_{2\to6}) + k_2\left(1 - \frac{L_{02}}{d_2}\right)(\boldsymbol{P}_{1\to4} \times \boldsymbol{P}_{3\to5}) = 0 \quad (1)$$

$$k_1\left(1 - \frac{L_{01}}{d_1}\right)(\boldsymbol{P}_{1\to2} \times \boldsymbol{P}_{2\to6}) + k_2\left(1 - \frac{L_{02}}{d_2}\right)(\boldsymbol{P}_{1\to3} \times \boldsymbol{P}_{3\to5}) = 0 \quad (2)$$

$$d_1^2 - \|\boldsymbol{P}_{2\to6}\|^2 = 0 \quad (3)$$

$$d_2^2 - \|\boldsymbol{P}_{3\to5}\|^2 = 0 \quad (4)$$

These equations are non-linear in the orientation angles γ_1, γ_2 which result in coupled polynomial equations in x_1, x_2, d_1, d_2 using tan half-angle substitutions where $x_i = \tan(\gamma_i/2)$.

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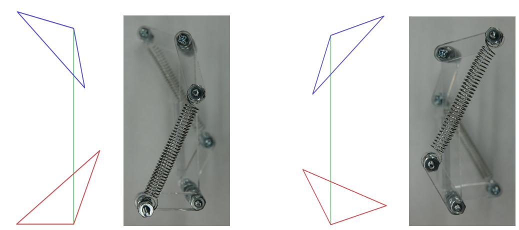
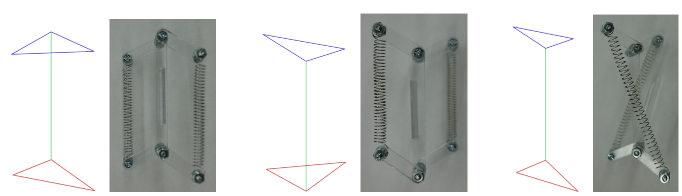
II. Multiple equilibria and shape change

Numerically, the equilibrium positions of such mechanisms are calculated numerical polynomial solver e.g. homotopy continuation-based Bertini Solver. Such non-linear mechanisms have multiple equilibrium positions and possess ability to move from one shape to another using active control of $L_{0i}, k_i \forall i = 1, 2$ and L_3. Interestingly, it is observed that as the free-lengths are varied, the number of equilibrium shapes varies from as many as 4 to 16. This is experimentally validated by constructing a model of the proposed mechanism using laser-cut acrylic, springs, nuts and bolts. The revolute joints at points $1, 2$ will be maintained by using bearings. Figure 2 illustrates five equilibrium positions where symmetry of spring elements and rigid body is preserved.



(a) Stable equilibrium positions



(b) Unstable equilibrium positions

Fig. 2: A planar CoTM displays multiple equilibria where two of the positions are stable and the other three are unstable.

III. Conclusion and demo

The CoTM mechanism can move from one stable shape to another by using active control of spring elements and rigid rod length. The demo will illustrate how shape change can be achieved through motor-cable-spring control (free-length).

References

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